Launch

Have students read the Why? section of the lesson. Ask:

- **Why is the volume of the planter related to the amount of soil needed to fill it?** Volume is the amount of space that a solid encloses. The space in the planter is filled with soil.
- **What are some other real-world applications of volume?** Sample answer: the volume of a cereal box affects the amount of cereal it contains; the volume of a building affects the size of heating or cooling system needed.

Teach

Ask the scaffolded questions for each example to build conceptual understanding for students at all levels.

1 Volume of Prisms

Example 1 Volume of a Prism

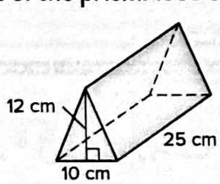
AL **If the height is changed to 8 centimeters, what is the new volume?** 480 cm^3

OL **What is the volume of a square prism with side length 7 centimeters and height 4 centimeters?** 196 cm^3

BL **A right triangular prism has legs 7.5 centimeters and 9 centimeters. If the volume of the prism is 270 cubic centimeters, what is the height of the prism?** 8 cm

Need Another Example?

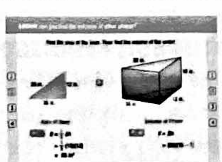
Find the volume of the prism. 1500 cm^3



Go Online!

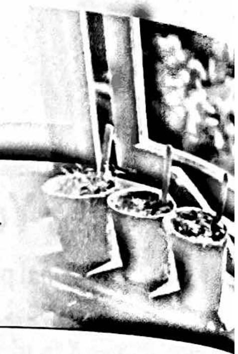
Interactive Whiteboard

Use the *eLesson, Lesson Presentation,* or *Interactive Classroom* to present this lesson.



LESSON 2
Volumes of Prisms and Cylinders

Then	Now	Why?
• You found surface areas of prisms and cylinders.	• 1 Find volumes of prisms. 2 Find volumes of cylinder.	• Planters come in a variety of shapes and sizes. You can approximate the amount of soil needed to fill a planter by finding the volume of the three-dimensional figure that it most resembles.



MP Mathematical Practices

1 Make sense of problems and persevere in solving them.

7 Look for and make use of structure.

Content Standards

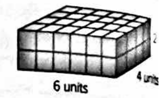
G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

G.GMD.3 Use volume formulas for cylinders, pyramids, cones and spheres to solve problems. ★

1 Volume of Prisms

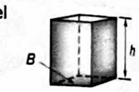
Recall that the volume of a solid is the measure of the amount of space the solid encloses. Volume is measured in cubic units.

The rectangular prism at the right has 6 · 4 or 24 cubic units in the bottom layer. Because there are two layers, the total volume is 24 · 2 or 48 cubic units.



Key Concept Volume of a Prism

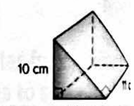
Words	The volume V of a prism is $V = Bh$, where B is the area of a base and h is the height of the prism.	Model
Symbols	$V = Bh$	



G.GMD.3

Example 1 Volume of a Prism

Find the volume of the prism.



Step 1 Find the area of the base B.

$B = \frac{1}{2}bh$ — Area of a triangle

$= \frac{1}{2}(12)(10)$ or 60 — $b = 12$ and $h = 10$

Step 2 Find the volume of the prism.

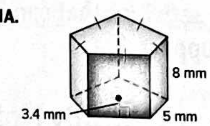
$V = Bh$ — Volume of a prism

$= 60(11)$ or 660 — $B = 60$ and $h = 11$

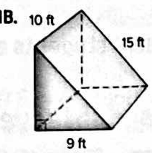
The volume of the prism is 660 cubic centimeters.

Guided Practice

1A. 340 mm^3



1B. 540 ft^2



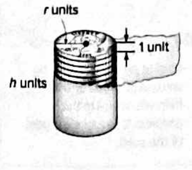
MP Mathematical Practices Strategies

Look for and make use of structure.
Help students understand the structure of prisms and cylinders. For example, ask:

- **How can you differentiate the base from the lateral sides of a prism or cylinder?** In a prism, the lateral sides are rectangles, but a base can be any polygon; in a cylinder, the lateral surface is curved and the base a circle.
- **Once the base is identified, what repeated reasoning is used to calculate volume?** Volume is simply multiple layers of the base one unit in thickness.

Go Online! Investigate volumes of prisms and cylinders by using the *Geometry Tools* in ConnectED.

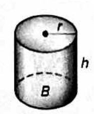
2 Volume of Cylinders Like a prism, the volume of a cylinder can be thought of as consisting of layers. For a cylinder, these layers are congruent circular discs, similar to the coins in the roll shown. If we interpret the area of the base as the volume of a one-unit-high layer and the height of the cylinder as the number of layers, then the volume of the cylinder is equal to the volume of a layer times the number of layers or the area of the base times the height.



Key Concept Volume of a Cylinder

Words The volume V of a cylinder is $V = Bh$ or $V = \pi r^2 h$, where B is the area of the base, h is the height of the cylinder, and r is the radius of the base.

Model

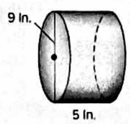


Symbols $V = Bh$ or $V = \pi r^2 h$

G.GMD.3

Example 2 Volume of a Cylinder

Find the volume of the cylinder at the right.



Estimate: $V \approx 3 \cdot 5^2 \cdot 5$ or 375 in^3

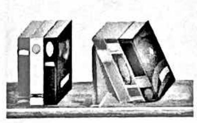
$V = \pi r^2 h$	Volume of a cylinder
$= \pi(4.5)^2(5)$	$r = 4.5$ and $h = 5$
≈ 318.1	Use a calculator.

The volume of the cylinder is about 318.1 cubic inches. This is fairly close to the estimate, so the answer is reasonable.

Guided Practice

2. Find the volume of a cylinder with a radius of 3 centimeters and a height of 8 centimeters. Round to the nearest tenth. 226.2 cm^2

The first group of books at the right represents a right prism. The second group represents an oblique prism. Both groups have the same number of books. If all the books are the same size, then the volume of both groups is the same.



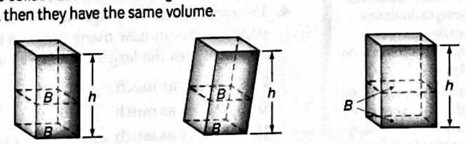
This demonstrates the following principle, which applies to all solids.

Watch Out
Cross-Sectional Area For solids with the same height to have the same volume, their cross sections must have the same area. The cross sections of the different solids do not have to be congruent polygons.

Key Concept Cavalieri's Principle

Words If two solids have the same height h and the same cross-sectional area B at every level, then they have the same volume.

Models



These prisms all have a volume of Bh.

Differentiated Instruction AL OL BL ELL

Kinesthetic Learners Have students take a tour of the school to find prisms of various shapes and sizes. Have them work with a partner to classify the shapes they find, and then calculate the lateral area and surface area of each prism. ELL

2 Volume of Cylinders

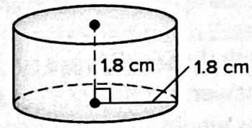
Example 2 Volume of a Cylinder

AL How do we know that the equations $V = Bh$ and $V = \pi r^2 h$ are equivalent for a cylinder? Because the base of a cylinder is a circle, and the area of the base B is equal to πr^2 for a circle, we know the two equations are equivalent.

OL If the radius of the cylinder is increased to 6 inches, what is the new volume? 565.5 in^3

BL If the volume of a cylinder with height 8 inches is 308 cubic inches, what is the diameter of the cylinder? about 7 in.

Need Another Example?
Find the volume of the cylinder. Round to the nearest tenth. 18.3 cm^3



Teaching Tips

Determining Height To help students see the difference between the height of the base of a triangular prism and the height of the prism, have them color the bases of the figures on their papers.

Area of Regular Polygons To review areas of regular polygons, see Lesson 10-4.

Watch Out!
Area and Volume Area is two-dimensional so it is measured in square units. Volume is three-dimensional so it is measured in cubic units.

Example 3 Volume of an Oblique Solid

AL Why is the height outside the prism? This is one way of visualizing the perpendicular height of the prism. If it were drawn from a difference vertex, it could show up inside the prism.

OL If the height of the prism is 8.9 centimeters, what is the volume? about 154 cm^3

BL If the volume of the prism with the same height is 131 cubic centimeters, what is the area of the base of the prism? about 20.5 cm^2

Formul: Sec 11.3
Volume of Pyramid / cones.

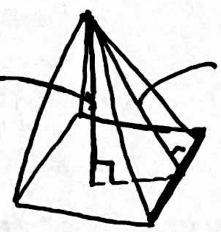
To Find Volume of a Pyramid &
Cone :

$$V = \frac{\text{Area base (height)}}{3} \quad \text{or} \quad \frac{Bh}{3}$$

B = Area of Base
h = height.

* Be careful with height (h) - It is the height of the Figure.

height

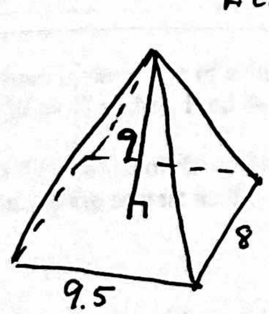


called slant height or lateral height
- Slant height is the height of the Triangle Face

- The height, slant height and segment from center of base to lateral Face form a RT Δ so that we can Find missing Parts

- Pyramids CAN have any shape for a base,
  Ex] Triangular Pyramids - Δ base
  Rectangular Pyramids - Rectangle base
  Hexagonal Pyramid - Hexagon base

EX]

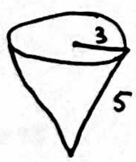


Find Volume:

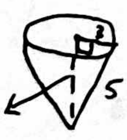
$$\frac{\text{Area base (height)}}{3} = \frac{\overbrace{9.5(8)}^{\text{Area base}}\ \overbrace{(9)}^{\text{height}}}{3}$$

$$= \frac{684}{3} = \boxed{228 = V}$$

EX]



Find Volume:



Draw on height

To Find height

$3^2 + h^2 = 5^2$

$9 + h^2 = 25$

$h^2 = 16$

$\boxed{h = 4}$

$$\text{Volume} = \frac{Bh}{3}$$

$$= \frac{\pi r^2 (h)}{3} = \frac{\pi(3^2)(5)}{3}$$

$$\boxed{= \frac{45\pi}{3} = 47.123}$$

11.4. - Surface Area / Volume Spheres.

Sphere:

Know as the Great circle (The circle that makes the figure spherical)

* The great circle is Just a circle. Remember Circumferenc $= 2\pi r$
Area $= \pi r^2$

Surface Area of a Sphere $= 4\pi r^2$

Surface Area of a Hemisphere $= \underbrace{\frac{4\pi r^2}{2}}_{\text{1/2 the sphere}} + \underbrace{\pi r^2}_{\text{The Exposed face}}$

* Remember Surface Area means the area of all outer faces. If you cut a sphere in half (hemisphere) the bottom face is now Exposed so we find the area of the whole sphere, cut it in half and then need to find the area of the Bottom exposed face.

Ex)

← Hemisphere

↑ Exposed face

Read through and See examples in Text Book. Pg 864-867

Volume of sphere $= \frac{4}{3}\pi r^3$. Please read through Book Pages for Explanation.

* I will supply these formulas for you on Test/Quiz.

Pg 822 10-25, 29, 30

Sec 11.6 Notes - ≅ & ~ Solids

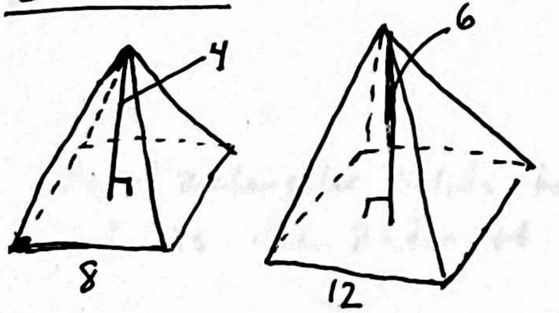
Similar (~) solids: Have same shape, <u>NOT</u> same size
- One is bigger/smaller than other.

If Solids are ~, then their corresponding linear measures such as height, Radius, side lengths HAVE <u>SAME</u> Ratio which also means we know the Scale Factor.

Scale Factor = Ratio of corresponding sides

Congruent (≅) solids: Have exactly <u>Same</u> Shape & Size.
- Their Scale Factor must be 1:1 or $\frac{1}{1}$

<u>Similar</u>



These are ~ because

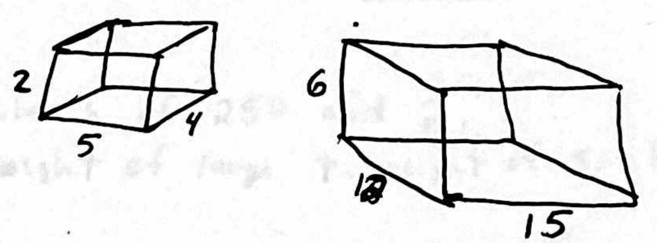
$\frac{4}{6} = \frac{2}{3}$

$\frac{8}{12} = 2/3$

corresponding lengths have Same Ratio

$\boxed{\text{Scale Factor} = \frac{2}{3}}$

<u>≅</u>



These are ≅ because

$\frac{2}{6} = \frac{1}{3}$

$\frac{5}{15} = \frac{1}{3}$

$\frac{4}{12} = \frac{1}{3}$

Same Ratio

$\boxed{\text{Scale Factor} = \frac{1}{3}}$

OVER →

Th^{m}: If 2 solids are $\sim$, then

1. The Ratio of their Scale Factor is $\frac{a}{b}$

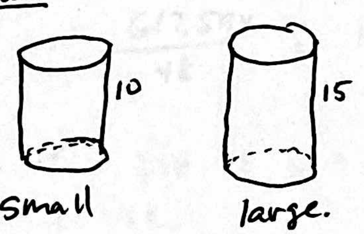
2. The Ratio of their areas is $\frac{a^2}{b^2}$

3. the Ratio of their volumes is $\frac{a^3}{b^3}$

a & b Represent corresponding side lengths.

Ex] Two $\sim$ cylinders have heights 10 & 15. what is Ratio of Volume from large To small cylinder

Answer



a = 10
b = 15

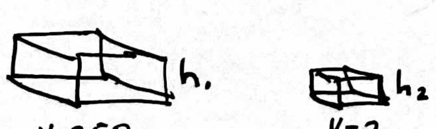
Ratio of volume

$= \frac{a^3}{b^3} \Rightarrow \frac{10^3}{15^3} = \frac{1000}{3375} = \frac{8}{27}$

* This is small To Big, we need Big : small so $\boxed{\frac{27}{8}}$

Ex]

Two Rectangular Solids have Volumes of 250 and 2. what is the Ratio of the height of large To height of small



Answer

Ratio of volume $= \frac{a^3}{b^3}$ If we Knew Just a & b. we know the side lengths. To Find a & b we must cube Root a & b To get Rid of 3 exponent.

Ratio of $V = \frac{250}{2} = \frac{a^3}{b^3}$

so $\frac{\sqrt[3]{250}}{\sqrt[3]{2}} = \frac{a}{b} = \frac{5}{1}$ } use Calculator To do $\sqrt[3]{}$

Formal sec 11.7

Density: measure of a Quantity of some physical Property per unit of Length, area or Volume.

Ex) Population Density: measurement of Population per unit of Area.

Density based on Area:

$$\text{Density} = \frac{\#\text{ of objects}}{\text{area}}$$

D = ratio of objects to Area

ex) Boston has a Pop of 617,594 with an area of 48 mi²

a) Find the Pop Density:

$$\frac{617,594}{48} = 12,866 \text{ Persons/mi}^2$$

b) Find the Pop of L.A if their Pop Density 8086.6. and their area is 469 mi²

$$D = \frac{\#\text{ objects}}{\text{area}} = 8086.6 = \frac{x}{469}$$

OVER

Pg 844

7-19